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1987 J. Phys. A: Math. Gen. 20 4383

(http://iopscience.iop.org/0305-4470/20/13/038)

J. Phys. A: Math. Gen. **20** (1987) 4383-4390. Printed in the UK

The supersymmetric formalism and the stochastic model symmetries in the fermion–fermion sector

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Received 25 July 1985, in final form 26 January 1987

Abstract. We consider a supersymmetric formulation of the two-point function and show explicitly that the underlying stochastic model symmetries can be implemented in the fermion-fermion sector through a compact parametrisation of the σ_F matrix.

1. Introduction

In recent years, the supersymmetric technique (SST) has been successfully applied to study various physical problems. In the theory of disordered systems as well as in the statistical theory of nuclear reactions, random Hamiltonians with Gaussian distributed elements are used to model statistical phenomena (see Efetov 1983, Wegner 1983, Verbaarschot *et al* 1985 and references therein). The physical quantities, defined as the energy averaged 'n-point functions', are obtained after an ergodicity assumption from the corresponding ensemble averaged 'n-point generating functions' which involve both commuting and anticommuting integration variables. Although many intrinsic properties and the general procedure of the SST have been already clarified, some aspects need to be better understood. A comprehensive and explicit discussion of the SST, applied to the calculation of compound-nucleus cross sections, was given by Verbaarschot, Weidenmüller and Zirnbauer (1985); one of the aspects extensively discussed by these authors, and with which we will be concerned here, is the problem of implementation of the underlying symmetries of the system in the parametrisation of the so-called composite variables (σ-matrix elements). This problem appears also in the 'replica trick' formulations for the two-point function where a hyperbolic symmetry was discovered and studied by Wegner (1979). Schäfer and Wegner (1980) showed that uniform convergence problems in the integration of the σ-variables are overcome when the matrix σ is parametrised in terms of the non-compact (hyperbolic) group of symmetry transformations and an appropriate shift of σ, consistent with the saddle-point structure, is performed. In the context of the supersymmetric formalism, the commuting composite variables form two classes: the variables of the 'boson-boson block' (σ_B matrix) and the variables of the 'fermion-fermion block' (σ_F matrix). Verbaarschot *et al* showed that, for a particular (later referred as the original) representation of the σ matrix, uniform convergence problems appear related to the σ_B variables. To deal with this problem, they turned their attention to the study of the

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group of symmetry transformations associated with the stochastic model and found that the transformations were non-compact with respect to both σ_B and σ_F (see appendix D of Verbaarschot *et al* (1985)). While this was desirable for σ_B, because the arguments of Schäfer and Wegner are applicable there, it is not for σ_F. The attempts to parametrise σ_F in terms of a non-compact group failed because of difficulties related to convergence and saddle-point properties. Therefore, they concluded that the parametrisation of the σ_F matrix 'is not consistent with the symmetries (of the stochastic model)... it being actually *impossible* to implement (those) symmetries in σ_F'. So, they were '*forced* to keep the original σ_F' which was later, after a 'compactification argument', represented in terms of a compact group. The understanding of this problem is the motivation of the present paper. It is certainly one aspect of the supersymmetric technique whose elucidation is very much called for.

We will show that, taking into account, consistently, conjugation and linear transformation properties of graded vectors, we obtain readily an explicit realisation of the group of symmetry transformations which contains a compact and a non-compact subgroup. Given these solutions and requiring invariant structure of the σ_F matrix obtained from the Hubbard-Stratonovitch transformations, we are left with the compact subgroup. In this way, we show that the symmetries of the stochastic model can be implemented in the parametrisation of σ_F. Problems of convergence and difficulties with respect to the structure of the saddle-point manifold disappear because it is not the hyperbolic but the elliptic symmetry which is associated with this sector. In § 2 we consider a two-point generating function, we calculate its ensemble average, discuss the Hubbard-Stratonovitch transformation and the convergence problems associated with it. In § 3 we restrict ourselves to the fermion-fermion sector, give explicit solutions for the group of symmetry transformations and construct and parametrise the matrix σ_F.

2. Two-point functions and symmetry properties

In this section we outline the relevant results and characteristic definitions for a supersymmetric formulation of the two-point function. We leave for the next section the problem of implementation of the stochastic model symmetries.

To keep contact with the work of Verbaarschot *et al*, let us consider the following two-point generating function

$$Z(\varepsilon_1, \varepsilon_2^*) = \frac{\det[\mathscr{D}_1(\varepsilon_1) \cdot \mathscr{D}_2^*(\varepsilon_2)]}{\det[\mathscr{D}_1'(\varepsilon_1) \cdot \mathscr{D}_2'^*(\varepsilon_2)]} \tag{2.1}$$

where, without loss of generality, the $N \times N$ matrices $\mathscr{D}_p$ and $\mathscr{D}_p'$ are written as

$$\begin{aligned}(\mathscr{D}_p)_{\mu\nu} &= \varepsilon_p \delta_{\mu\nu} - H_{\mu\nu} + J_{\mu\nu} \\ (\mathscr{D}_p')_{\mu\nu} &= \varepsilon_p \delta_{\mu\nu} - H_{\mu\nu} + J'_{\mu\nu}\end{aligned} \tag{2.2}$$

being $\operatorname{Im} \varepsilon_1 = \operatorname{Im} \varepsilon_2 > 0$ and the Hamiltonian matrix elements $H_{\mu\nu}$ are members of an ensemble of Gaussian distributed variables centred at the origin with variance

$$\overline{H_{\mu\nu} H_{\mu'\nu'}} = \frac{\lambda^2}{N} (\delta_{\mu\mu'} \delta_{\nu\nu'} + \delta_{\mu\nu'} \delta_{\nu\mu'}). \tag{2.3}$$

$J_{\mu\nu}$ and $J'_{\mu\nu}$ are source terms defined according to the physical problem. When $J_{\mu\nu} = J'_{\mu\nu}$, the generating function is normalised to unity.

Using integration properties of the anticommuting variables and the graded matrices

$$\psi=\begin{pmatrix}\varphi_c\\ \varphi_a\end{pmatrix}\qquad \varphi_c=\begin{pmatrix}s\\ t\\ u\\ v\end{pmatrix}\qquad \varphi_a=\begin{pmatrix}\chi\\ \chi^{x}\\ \eta\\ \eta^{x}\end{pmatrix}\tag{2.4}$$

$$\Delta=\begin{pmatrix}\mathscr{D} & 0\\ 0 & \mathscr{D}'\end{pmatrix}\qquad \mathscr{D}=\begin{pmatrix}\mathscr{D}_1 & 0 & & 0\\ 0 & \mathscr{D}_2 & & \\ & & \mathscr{D}_2^* & 0\\ 0 & & 0 & \mathscr{D}_2^*\end{pmatrix}\tag{2.5}$$

where s, t, u and v are column vectors of dimension N whose elements are real commuting (ordinary) variables and χ, χ^{x}, η and η^{x} vectors of dimension N whose elements are anticommuting (Grassmann) variables, we can represent the generating function in the following way:

$$Z(\varepsilon_1,\varepsilon_2^*)=\int \mathrm{d}[\psi]\exp(\tfrac{1}{2}\mathrm{i}\psi^{\ddagger}\Delta\psi)\tag{2.6}$$

where the imaginary number i is added to assure convergence for the integration over the commuting variables and the symbol ‡ is used to denote the adjoint operation of the second kind (see Rittenberg and Scheunert (1978)). We separate the matrix Δ as follows:

$$\Delta=\mathcal{N}(\varepsilon_1,\varepsilon_2,J,J')+L_8\otimes H\tag{2.7}$$

where L is a diagonal 8×8 graded matrix whose elements are 1, 1, -1, -1, 1, 1, -1, -1. For the ensemble average of the generating function we obtain

$$\begin{aligned}\overline{Z(\varepsilon_1,\varepsilon_2^*)}&=\int \mathrm{d}[\psi]\exp(\tfrac{1}{2}\mathrm{i}\psi^{\ddagger}\mathcal{N}\psi)\\ &\quad\times\exp\bigg(-\frac{\lambda^2}{4N}\sum_{\mu,\nu=1}^{N}(s_\mu s_\nu+t_\mu t_\nu-u_\mu u_\nu-v_\mu v_\nu\\ &\quad+\chi_\mu^{x}\chi_\nu-\chi_\mu\chi_\nu^{x}-\eta_\mu^{x}\eta_\nu+\eta_\mu\eta_\nu^{x})^2\bigg).\end{aligned}\tag{2.8}$$

The last exponential factor carries the symmetries of H. Defining the matrix (the superscript T means transposition)

$$\phi=\begin{pmatrix}s^{\mathrm{T}}\\ t^{\mathrm{T}}\\ u^{\mathrm{T}}\\ v^{\mathrm{T}}\\ \chi^{\mathrm{T}}\\ \chi^{\ddagger}\\ \eta^{\mathrm{T}}\\ \eta^{\ddagger}\end{pmatrix}\qquad \phi^{\ddagger}=(s,t,u,v,\chi^{x},-\chi,\eta^{x},-\eta)\tag{2.9}$$

we write (2.8) as

$$\overline{Z(\varepsilon_1,\varepsilon_2^*)}=\int \mathrm{d}[\psi]\exp\bigg(\tfrac{1}{2}\mathrm{i}\psi^{\ddagger}\mathcal{N}\psi-\frac{\lambda^2}{4N}\operatorname{trg}(L^{1/2}\phi\phi^{\ddagger}L^{1/2})^2\bigg).\tag{2.10}$$

Here, trg denotes as usual the graded trace. At this level, one can, in principle, obtain the physical quantities as derivatives of the averaged generating function. However, we have still a huge number of integration variables ($8N$ variables, with $N \to \infty$ in nuclear reaction problems). The reduction of this number is thus the main objective. Due to the quartic order terms in ϕ. The ψ integration cannot be managed as it stands. A normal procedure to overcome this problem is to have recourse to the Hubbard-Stratonovitch transformation

$$\exp\left(-\frac{\lambda^2}{4N}\operatorname{trg}(L^{1/2}\phi\phi^{\ddagger}L^{1/2})^2\right) = \frac{1}{2}\left(\frac{N}{\pi}\right)^3 \int \mathrm{d}[\sigma] \exp\left(-\frac{N}{4}\operatorname{trg}\sigma^2 + \frac{i\lambda}{2}\operatorname{trg}(\sigma L^{1/2}\phi\phi^{\ddagger}L^{1/2})\right). \tag{2.11}$$

This transformation introduces new integration variables (the independent elements of the graded 8×8 matrix σ) but we are left with only quadratic terms in the ψ variables. The integration over the $8N$ ψ variables can be done provided the σ integration is uniformly convergent with respect to ψ, i.e. when the interchange of the integrations over $\mathrm{d}[\sigma]$ and $\mathrm{d}[\psi]$ is possible. This is a mathematical problem. We have also a physical problem that stems from the fact that the particular structure of the σ matrix must be consistent with the shift $\sigma \to \sigma + (\mathrm{i}\lambda/N)L^{1/2}\phi\phi^{\ddagger}L^{1/2}$, implicit in (2.11). In other words, the physical information embodied by the symmetries of the form $L^{1/2}\phi\phi^{\ddagger}L^{1/2}$ should be preserved and carried by the σ matrix. This is, after all, the idea of the implementation of such symmetries in the parametrisation of the matrix σ.

The group of symmetry transformations $\hat{T}: \phi \to \hat{T}\phi$ is determined from the condition that it leaves invariant the form $L^{1/2}\phi\phi^{\ddagger}L^{1/2}$ and from the definition of charge conjugation for anticommuting variables ($(\chi_\mu^{x})^{x} = -\chi_\mu$). Therefore

$$\hat{T}^{\ddagger}L\hat{T} = L \tag{2.12}$$

and

$$\hat{T}^{x} = C^{\mathrm{T}}\hat{T}C \tag{2.13}$$

with

$$C = \begin{pmatrix} I_4 & 0 \\ 0 & \Gamma \end{pmatrix} \qquad \Gamma = \left(\begin{array}{cc|cc} 0 & -1 & & \\ 1 & 0 & \multicolumn{2}{c}{0} \\ \hline & & 0 & -1 \\ \multicolumn{2}{c|}{0} & 1 & 0 \end{array}\right). \tag{2.14}$$

Here, I_m is the $m\times m$ unit matrix. It is easy to see that the transformation $\hat{T}$ induces the following transformation for the composite variables:

$$\sigma \to T^{-1}\sigma T \tag{2.15}$$

where

$$T = L^{1/2}\hat{T}L^{-1/2} \qquad T^{-1} = L^{-1/2}\hat{T}^{\ddagger}L^{1/2}. \tag{2.16}$$

Clearly

$$T^{-1} = C^{\mathrm{T}}T^{\mathrm{T}}C. \tag{2.17}$$

The group of transformations $\hat{T}$, known as the pseudo-unitary orthosymplectic Lie group UOSP(2, 2/2, 2), has been studied by Verbaarschot *et al* in terms of its generators. These authors noted the non-compact character of the transformations for σ_{B} and σ_{F}. In the next section, we show explicitly that the compact solutions are also genuine solutions of (2.17).

3. Parametrisation of the σ_F matrix

Since we are concerned only with the fermion–fermion sector, let us take the commuting variables of ψ equal to zero. This is precisely the kind of generating function that we would have in a replica formalism for the two-point function with anticommuting variables. In this case, our generating function (with the obvious reductions of $\mathcal{N}$, ϕ and L) takes the form

$$\overline{Z(\varepsilon, \varepsilon_2^*)} = \int \mathrm{d}[\varphi_a] \exp\left(\frac{\mathrm{i}}{2}\varphi_a^\ddagger \mathcal{N}_F \varphi_a - \frac{\lambda^2}{4N}\operatorname{Tr}(L_F^{1/2}\phi_a\phi_a^\ddagger L_F^{1/2})^2\right) \tag{3.1}$$

and the elements of the group of symmetry transformations satisfy the relation

$$t^{-1}t = I_4 \tag{3.2}$$

where

$$t^{-1} = (\Gamma^{\mathrm{T}} t \Gamma)^{\mathrm{T}}. \tag{3.3}$$

According to these requirements, if

$$t_k = \begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{pmatrix} \tag{3.4}$$

is one element of the group, then its inverse defined by equation (3.3) will have the structure

$$t_k^{-1} = \begin{pmatrix} t_{22} & -t_{12} & t_{42} & -t_{32} \\ -t_{21} & t_{11} & -t_{41} & t_{31} \\ t_{24} & -t_{14} & t_{44} & -t_{34} \\ -t_{23} & t_{13} & -t_{43} & t_{33} \end{pmatrix}. \tag{3.5}$$

We want to emphasise here that a careless use and interpretation of the adjoint operation in the presence of anticommuting variables, could give rise to non-compatible solutions for the group of symmetry transformations. We now give the following explicit solutions to (3.2) and (3.3):

$$t_1 = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\alpha}\cosh\omega & 0 & -\mathrm{e}^{-\mathrm{i}\theta}\sinh\omega & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\alpha}\cosh\omega & 0 & \mathrm{e}^{\mathrm{i}\theta}\sinh\omega \\ -\mathrm{e}^{\mathrm{i}\theta}\sinh\omega & 0 & \mathrm{e}^{\mathrm{i}\alpha}\cosh\omega & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta}\sinh\omega & 0 & \mathrm{e}^{-\mathrm{i}\alpha}\cosh\omega \end{pmatrix} \tag{3.6}$$

$$t_2 = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\beta}\cosh z & 0 & 0 & -\mathrm{e}^{-\mathrm{i}\gamma}\sinh z \\ 0 & \mathrm{e}^{\mathrm{i}\beta}\cosh z & -\mathrm{e}^{\mathrm{i}\gamma}\sinh z & 0 \\ 0 & -\mathrm{e}^{\mathrm{i}\gamma}\sinh z & \mathrm{e}^{\mathrm{i}\beta}\cosh z & 0 \\ -\mathrm{e}^{-\mathrm{i}\gamma}\sinh z & 0 & 0 & \mathrm{e}^{-\mathrm{i}\beta}\cosh z \end{pmatrix} \tag{3.7}$$

$$t_3 = \begin{pmatrix} \begin{matrix} \mathrm{e}^{\mathrm{i}\varphi}\cos v & \mathrm{e}^{\mathrm{i}\psi}\sin v \\ \mathrm{e}^{-\mathrm{i}\psi}\sin v & \mathrm{e}^{-\mathrm{i}\varphi}\cos v \end{matrix} & 0 \\ 0 & \begin{matrix} \mathrm{e}^{\mathrm{i}\varphi'}\cos v' & \mathrm{e}^{\mathrm{i}\psi'}\sin v' \\ \mathrm{e}^{-\mathrm{i}\psi'}\sin v' & \mathrm{e}^{-\mathrm{i}\varphi'}\cos v' \end{matrix} \end{pmatrix} \tag{3.8}$$

where all the 'latitude angles' lie in the interval $[-\pi/2, \pi/2]$, the parametric space of the longitudinal angles v and v' is $[-\pi, \pi)$ and ω and z are complex variables whose real parts are non-compact parameters and their imaginary parts are compact parameters lying in the interval $[-\pi, \pi)$. The inverses are defined according to (3.5), for example

$$t_1^{-1} = \begin{pmatrix} e^{i\alpha}\cosh\omega & 0 & e^{-i\theta}\sinh\omega & 0 \\ 0 & e^{-i\alpha}\cosh\omega & 0 & -e^{i\theta}\sinh\omega \\ e^{i\theta}\sinh\omega & 0 & e^{-i\alpha}\cosh\omega & 0 \\ 0 & -e^{-i\theta}\sinh\omega & 0 & e^{i\alpha}\cosh\omega \end{pmatrix}. \tag{3.9}$$

It is trivial to see that each of these subgroups and therefore the full group contains a compact and a non-compact subgroup.

Before we come to the parametrisation problem, let us explicitly construct the matrix σ_F. Given the matrices

$$L_F^{1/2} = \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & & 0 \\ & 0 & i & 0 \\ & & 0 & i \end{pmatrix} \tag{3.10}$$

$$\phi_a\phi_a^\ddagger = \begin{pmatrix} \chi^T\chi^\times & 0 & \chi^T\eta^\times & -\chi^T\eta \\ 0 & -\chi^\ddagger\chi & \chi^\ddagger\eta^\times & -\chi^\ddagger\eta \\ \eta^T\chi^\times & -\eta^T\chi & \eta^T\eta^\times & 0 \\ \eta^\ddagger\chi^\times & -\eta^\ddagger\chi & 0 & -\eta^\ddagger\eta \end{pmatrix} \tag{3.11}$$

we have that

$$\mathrm{Tr}(L_F^{1/2}\phi_a\phi_a^\ddagger L_F^{1/2})^2 = 2(\chi^\ddagger\chi)^2 - 4\chi^\ddagger\eta\eta^\ddagger\chi + 4\chi^\ddagger\eta^\times\eta^T\chi + 2(\eta^\ddagger\eta)^2. \tag{3.12}$$

Using Gaussian integrals like

$$\exp\left(-\frac{\lambda^2}{2N}(\chi^\ddagger\chi)^2\right) = \left(\frac{N}{2\pi}\right)^{1/2} \int \mathrm{d}r\, \exp\left(-\frac{N}{2}r^2 - i\lambda r\chi^\ddagger\chi\right) \tag{3.13}$$

$$\exp\left(\frac{\lambda^2}{4N}(\chi^\ddagger\eta^\times + \eta^T\chi)^2\right) = \left(\frac{N}{\pi}\right)^{1/2} \int \mathrm{d}p\, \exp[-Np^2 \pm \lambda p(\chi^\ddagger\eta^\times + \eta^T\chi)] \tag{3.14}$$

and

$$\exp\left(-\frac{\lambda^2}{4N}(\chi^\ddagger\eta^\times - \eta^T\chi)^2\right) = \left(\frac{N}{\pi}\right)^{1/2} \int \mathrm{d}q\, \exp[-Nq^2 \mp i\lambda q(\chi^\ddagger\eta^\times - \eta^T\chi)] \tag{3.15}$$

we obtain the following relation:

$$\exp\left(-\frac{\lambda^2}{2N}[(\chi^\ddagger\chi)^2 + (\eta^\ddagger\eta)^2 - 2\chi^\ddagger\eta\eta^\ddagger\chi + 2\chi^\ddagger\eta^\times\eta^T\chi]\right)$$

$$= \frac{N^3}{2\pi^3}\int \mathrm{d}r\,\mathrm{d}r'\,\mathrm{d}p\,\mathrm{d}q\,\mathrm{d}p'\,\mathrm{d}q'\, \exp\left(-\frac{N}{4}[2r^2 + 2r'^2 + 4(p^2+q^2+p'^2+q'^2)]\right.$$

$$- i\lambda[r\chi^\ddagger\chi + r'\eta^\ddagger\eta + i(p-iq)\chi^\ddagger\eta - i(p+iq)\eta^\ddagger\chi$$

$$-\mathrm{i}(p'+\mathrm{i}q')\chi^{\ddagger}\eta^{\times}+\mathrm{i}(p'-\mathrm{i}q')\eta^{\mathrm{T}}\chi]\Big). \tag{3.16}$$

With

$$\sigma_{\mathrm{F}}=\begin{pmatrix} r & 0 & z^* & -y \\ 0 & r & y^* & z \\ z & y & r' & 0 \\ -y^* & z^* & 0 & r' \end{pmatrix} \tag{3.17}$$

where

$$z=p+\mathrm{i}q \qquad y=p'+\mathrm{i}q' \tag{3.18}$$

the last integral equation can be written as

$$\exp\left(-\frac{\lambda^2}{4N}\operatorname{Tr}(L_{\mathrm{F}}^{1/2}\phi_a\phi_a^{\ddagger}L_{\mathrm{F}}^{1/2})^2\right)$$
$$=\frac{N^3}{2\pi^3}\int \mathrm{d}[\sigma_{\mathrm{F}}]\exp\left(-\frac{N}{4}\operatorname{Tr}\sigma_{\mathrm{F}}^2+\frac{\mathrm{i}\lambda}{2}\operatorname{Tr}(\sigma_{\mathrm{F}}L_{\mathrm{F}}^{1/2}\phi_a\phi_a^{\ddagger}L_{\mathrm{F}}^{1/2})\right). \tag{3.19}$$

The matrix σ_{F} that we obtain here is exactly the same as the one given for this sector by Verbaarschot *et al*, except that an overall factor i appears there to compensate the change of sign coming from the graded trace. This factor is irrelevant and has no effect on our discussion.

Now we concentrate on the parametrisation problem. To simplify the analysis, let us write the matrix σ_{F} as

$$\sigma_{\mathrm{F}}=\begin{pmatrix} rI_2 & \mu U \\ \mu U^+ & r'I_2 \end{pmatrix}=\begin{pmatrix} U & 0 \\ 0 & I_2 \end{pmatrix}\Omega_{\mathrm{F}}\begin{pmatrix} U^+ & 0 \\ 0 & I_2 \end{pmatrix} \tag{3.20}$$

where r, r' and μ are real parameters, U is a 2×2 special unitary matrix and

$$\Omega_{\mathrm{F}}=\begin{pmatrix} r & \mu \\ \mu & r' \end{pmatrix}\otimes I_2. \tag{3.21}$$

This matrix has two different real eigenvalues. All that we want to know is the matrix t_0 such that

$$\Omega_{\mathrm{F}}=t_0^{-1}\begin{pmatrix} e_1 & 0 & & \\ 0 & e_1 & & 0 \\ & & e_2 & 0 \\ 0 & & 0 & e_2 \end{pmatrix}t_0\equiv t_0^{-1}\sigma_{\mathrm{D}}t_0. \tag{3.22}$$

It is easy to see that such a matrix has to be one belonging to the subgroup t_1. Taking into account the structure of Ω_{F}, we fix the values $\alpha+\theta=\pi/2$, $\omega=\mathrm{i}x$. Finally, choosing $\alpha=0$, we have for σ_{F} the following compact parametrisation:

$$\sigma_{\mathrm{F}}=\begin{pmatrix} U & 0 \\ 0 & I_2 \end{pmatrix}t_0^{-1}\sigma_{\mathrm{D}}t_0\begin{pmatrix} U^+ & 0 \\ 0 & I_2 \end{pmatrix} \tag{3.23}$$

where

$$t_0=\begin{pmatrix} \cos x & 0 & -\sin x & 0 \\ 0 & \cos x & 0 & -\sin x \\ \sin x & 0 & \cos x & 0 \\ 0 & \sin x & 0 & \cos x \end{pmatrix} \qquad t_0^{-1}=\Gamma^{\mathrm{T}}t_0^{\mathrm{T}}\Gamma. \tag{3.24}$$

Formally, once the interchange of the integrations over $d[\sigma]$ and $d[\psi]$ is justified, the ψ integration can be trivially performed. The problem is reduced into a non-linear σ model and one proceeds as usual. The reader is referred to the paper of Verbaarschot *et al*, where two-point functions (compound nucleus cross sections) are calculated and expressed as threefold integrals.

4. Conclusion

In this quite direct and simple way we have shown that the group of symmetry transformations associated with the 'fermion–fermion sector' always contains a compact and a non-compact subgroup and that the underlying symmetries of the physical system are implemented by a compact parametrisation of the σ_F matrix. This conclusion holds also for time-reversal non-invariant systems.

Acknowledgments

I wish to thank Professor H A Weidenmüller for encouraging me to write the present paper. I am indebted to J J M Verbaarschot, H A Weidenmüller and M R Zirnbauer for making available their manuscript prior to publication.

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